NAG Fortran Library Routine Document

S21CAF

Note: before using this routine, please read the Users' Note for your implementation to check the interpretation of ***bold italicised*** terms and other implementation-dependent details.

1 Purpose

S21CAF evaluates the Jacobian elliptic functions sn, cn and dn.

2 Specification

```
SUBROUTINE S21CAF(U, M, SN, CN, DN, IFAIL)
INTEGER           IFAIL
real              U, M, SN, CN, DN
```

3 Description

This routine evaluates the Jacobian elliptic functions of argument u and parameter m,

$$\begin{aligned} \mathrm{sn}(u|m) &= \sin\phi, \\ \mathrm{cn}(u|m) &= \cos\phi, \\ \mathrm{dn}(u|m) &= \sqrt{1 - m\sin^2\phi}, \end{aligned}$$

where ϕ, called the *amplitude* of u, is defined by the integral

$$u = \int_0^{\phi} \frac{d\theta}{\sqrt{1 - m\sin^2\theta}}.$$

The elliptic functions are sometimes written simply as sn$u$, cn$u$ and dn$u$, avoiding explicit reference to the parameter m.

Another nine elliptic functions may be computed via the formulae

$$\begin{aligned} \mathrm{cd}\,u &= \mathrm{cn}\,u/\mathrm{dn}\,u \\ \mathrm{sd}\,u &= \mathrm{sn}\,u/\mathrm{dn}\,u \\ \mathrm{nd}\,u &= 1/\mathrm{dn}\,u \\ \mathrm{dc}\,u &= \mathrm{dn}u/\mathrm{cn}\,u \\ \mathrm{nc}\,u &= 1/\mathrm{cn}\,u \\ \mathrm{sc}\,u &= \mathrm{sn}\,u/\mathrm{cn}\,u \\ \mathrm{ns}\,u &= 1/\mathrm{sn}\,u \\ \mathrm{ds}\,u &= \mathrm{dn}\,u/\mathrm{sn}\,u \\ \mathrm{cs}\,u &= \mathrm{cn}\,u/\mathrm{sn}\,u \end{aligned}$$

(see Abramowitz and Stegun (1972)).

S21CAF is based on a procedure given by Bulirsch (1960), and uses the process of the arithmetic-geometric mean (16.9 in Abramowitz and Stegun (1972)). Constraints are placed on the values of u and m in order to avoid the possibility of machine overflow.

4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Bulirsch R (1960) Numerical calculation of elliptic integrals and elliptic functions *Numer. Math.* **7** 76–90

5 Parameters

1: U – ***real*** *Input*
2: M – ***real*** *Input*

On entry: the argument u and the parameter m of the functions, respectively.

Constraints:

ABS(U) $\leq \sqrt{\lambda}$, where $\lambda = 1/$X02AMF,
ABS(M) $\leq \sqrt{\lambda}$ if ABS(U) $< 1/\sqrt{\lambda}$.

3: SN – ***real*** *Output*
4: CN – ***real*** *Output*
5: DN – ***real*** *Output*

On exit: the values of the functions $\mathrm{sn}\,u$, $\mathrm{cn}\,u$ and $\mathrm{dn}\,u$, respectively.

6: IFAIL – INTEGER *Input/Output*

On entry: IFAIL must be set to 0, -1 or 1. Users who are unfamiliar with this parameter should refer to Chapter P01 for details.

On exit: IFAIL $= 0$ unless the routine detects an error (see Section 6).

For environments where it might be inappropriate to halt program execution when an error is detected, the value -1 or 1 is recommended. If the output of error messages is undesirable, then the value 1 is recommended. Otherwise, for users not familiar with this parameter the recommended value is 0. **When the value -1 or 1 is used it is essential to test the value of IFAIL on exit.**

6 Error Indicators and Warnings

If on entry IFAIL $= 0$ or -1, explanatory error messages are output on the current error message unit (as defined by X04AAF).

Errors or warnings detected by the routine:

IFAIL $= 1$

On entry, ABS(U) $> \sqrt{\lambda}$, where $\lambda = 1/$X02AMF( ).

IFAIL $= 2$

On entry, ABS(M) $> \sqrt{\lambda}$ and ABS(U) $< 1/\sqrt{\lambda}$.

7 Accuracy

In principle the routine is capable of achieving full relative precision in the computed values. However, the accuracy obtainable in practice depends on the accuracy of the Fortran intrinsic functions for elementary functions such as SIN and COS.

8 Further Comments

None.

9 Example

The example program reads values of the argument u and parameter m from a file, evaluates the function and prints the results.

9.1 Program Text

Note: the listing of the example program presented below uses ***bold italicised*** terms to denote precision-dependent details. Please read the Users' Note for your implementation to check the interpretation of these terms. As explained in the Essential Introduction to this manual, the results produced may not be identical for all implementations.

```
*     S21CAF Example Program Text
*     Mark 15 Release. NAG Copyright 1991
*     .. Parameters ..
      INTEGER          NIN, NOUT
      PARAMETER        (NIN=5,NOUT=6)
*     .. Local Scalars ..
      real             CN, DN, M, SN, U
      INTEGER          IFAIL
*     .. External Subroutines ..
      EXTERNAL         S21CAF
*     .. Executable Statements ..
      WRITE (NOUT,*) 'S21CAF Example Program Results'
*     Skip heading in data file
      READ (NIN,*)
      WRITE (NOUT,*)
      WRITE (NOUT,*)
     + '          U            M           SN           CN           DN'
   20 READ (NIN,*,END=40) U, M
*
      IFAIL = 0
      CALL S21CAF(U,M,SN,CN,DN,IFAIL)
*
      WRITE (NOUT,99999) U, M, SN, CN, DN
      GO TO 20
   40 STOP
*
99999 FORMAT (3X,5e13.4)
      END
```

9.2 Program Data

```
S21CAF Example Program Data
  0.2   0.3
  5.0  -1.0
 -0.5  -0.1
 10.0  11.0
```

9.3 Program Results

```
 S21CAF Example Program Results

          U            M           SN           CN           DN
      0.2000E+00   0.3000E+00   0.1983E+00   0.9801E+00   0.9941E+00
      0.5000E+01  -0.1000E+01  -0.2440E+00   0.9698E+00   0.1029E+01
     -0.5000E+00  -0.1000E+00  -0.4812E+00   0.8766E+00   0.1012E+01
      0.1000E+02   0.1100E+02   0.2512E+00   0.9679E+00   0.5528E+00
```